
I

ELECTROMAGNETIC ABSORPTION BY ROCKS
WITH SOME EXPERIMENTAL OBSERVATIONS
TAKEN AT THE MAMMOTH CAVE OF KENTUCKY

II

MODIFICATION OF THE TORSION BALANCE

By

J. WALLACE JOYCE

A Dissertation submitted to the Board of
University Studies in conformity with the re-
quirement for the degree of Doctor of Philosophy.

1931
THE JOHNS HOPKINS UNIVERSITY
BALTIMORE, MARYLAND

U. S. Department of Commerce, Bureau of Mines,
Washington, D. C., Technical Paper 497, 1931, and
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VITA

James Wallace Joyce, son of James and Annie Malkin Joyce, was born in Providence, Rhode Island, July 8, 1907.

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During the summers of 1928, 1929, and 1930 he was engaged in experimental geophysical investigations in various parts of this country as an employee of the United States Bureau of Mines.



U. S. DEPARTMENT OF COMMERCE

R. P. LAMONT, Secretary

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ELECTROMAGNETIC ABSORPTION BY ROCKS, WITH SOME EXPERIMENTAL OBSERVATIONS TAKEN AT THE MAMMOTH CAVE OF KENTUCKY¹

By J. WALLACE JOYCE²

INTRODUCTION

This paper endeavors to shed some light on the question of the penetration of electromagnetic fields or waves into the ground. It is intended to settle definitely the fact that such waves penetrate the earth's surface and that, in so doing, absorption occurs. The question is vital to geophysics, since the successful operation of all induction methods depends upon this factor. That the electromagnetic field weakens as it spreads itself out into an ever-increasing volume of space is quite generally admitted without much comment. However, in connection with the loss of energy of the field due to the medium itself, there is need for investigation. If, in certain instances, the amount of this absorption is sufficient to screen conducting material beyond from the electromagnetic field so as to render this method of prospecting inefficient it should be known and accounted for.

Various opinions and suppositions have been offered in an attempt to settle this matter. Much experimental work has been done but no really conclusive evidence has been presented up to the present time. The theoretical discussions in some instances differ so greatly among themselves that there is always a doubt left in the reader's mind as to the applicability of the formula. In addition, attenuating circumstances under which experimental evidence was obtained often threw grave doubts upon the conclusiveness of the results. The influence of such factors, for example, as metal conductors or straight-air columns leading to the surface raised the question of the modification of transmission introduced by these agencies. Some believed firmly that transmission occurred primarily through the rock itself, but there was no real conclusive evidence to substantiate their opinion. On the other hand, the necessity of hermetically sealing an oscillating circuit to prevent leakage to the outside and the ease with which metallic conductors serve as transmitters of electrical waves presented very real arguments for those expressing contrary opinions.

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² Assistant geophysical technologist, U. S. Bureau of Mines.

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The Bureau of Mines also wishes to express its indebtedness to the trustees of the Mammoth Cave estate for making this work possible and to M. L. Charlet, general manager of the Mammoth Cave.

FUNDAMENTAL PRINCIPLES OF INDUCTION METHODS

Briefly, the principles involved in all electrical induction methods are as follows: An alternating electromagnetic field is established in the region to be investigated, which is generally at the surface of the ground. This is accomplished by using either a vertical or a horizontal loop excited with an alternating current. The electromagnetic field from this loop reacts upon any conducting material present and, in accordance with the laws of mutual induction, establishes currents therein. These currents in turn have an alternating field associated with them which combines with the field of the loop to give a resultant field.

If no conducting material was present in the region under the influence of the exciting loop there would be no secondary field, and the configuration of the exciting field would correspond to that predicted by theory for the ideal case of a loop in a homogeneous medium. However, any secondary fields introduced by currents flowing in conducting materials will cause anomalies in the field at the surface, and a study of these is of great assistance in predicting the location of the disturbing conductor.

FACTORS INFLUENCING REACTION OF CONDUCTING BODIES

There are many factors which influence the reaction a conducting body will produce at the surface. Some of the most important are enumerated below.

First, there is the effect of depth or distance from the exciting loop, aside from any phenomena introduced by the ground itself. It must be remembered that both the exciting and secondary fields become weaker as the distances from their sources increase. For current paths which are small relative to their distances from the sources the inverse cube of the distance enters as a controlling factor. This inverse cube effect enters twice into the operation, first from the exciting loop or primary to the conducting body in the earth and then from the conducting body back again to the surface. From this it is quite apparent that the factor of depth assumes an important and often controlling rôle in the magnitude of the anomalies that may be observed at the surface.

Second, there is the impedance or resistance to the flow of current of the buried material. The lower this impedance the greater will be the current in the conducting body and the larger the anomaly caused by it in the primary field at the surface of the ground. A low impedance generally implies low resistance, but at the higher frequencies the inductance or electrical inertia serves to increase this impedance. To avoid this it is the usual practice to employ rather low frequencies. Those of the order of 500 cycles per second have been found to be quite effective. The resistivity will in general

depend on the structure of the material itself. Usually the more mineral in the ore the lower its resistivity, except where dissemination occurs. In that case the degree of dissemination also becomes an important factor, since it prevents a continuous conducting circuit.

It is quite important that an appreciable difference exists between the resistivity of the structure and that of the surrounding material. The greater this difference the more the secondary currents will be concentrated in the structure, under favorable excitation conditions, and the sharper will be the anomalies at the surface.

A third factor is the absorption of the magnetic and electric vectors brought about by the intervening material between the conductor and the exciting loop. This quantity will in general be a function of the frequency, the thickness of the rock or intervening material and its resistivity, dielectric constant, and permeability.

This paper deals with an investigation of this absorption phenomenon and endeavors to correlate experimental investigations with the theoretical formulas.

ABSORPTION FORMULAS

As a theoretical postulate upon which to build experimental evidence several of the prominent absorption formulas are now to be presented and discussed. All apply to plain electromagnetic waves in a homogeneous medium. For the convenience of the reader the following symbols will be used throughout the paper:

- f =frequency in cycles per second.
- ϵ =dielectric constant of the structure.
- ρ =resistivity of the structure.
- δ =specific conductivity of the structure.
- μ =permeability of the structure.
- l =depth or thickness of the structure.
- c =velocity of light in vacuum=3 times 10^{10} centimeters per second.
- $\alpha=\alpha_1+j\alpha_2$ is the attenuation constant of the wave, of which
- α_1 =attenuation in amplitude, or magnitude of vector.
- α_2 =attenuation in phase or slipping in speed due to the electrical constants of the structure.
- $j=\sqrt{-1}$ Gaussian operator.
- δ =absorption coefficient or decrease of the strength of the field.
- $e=2.718$ base of Napierian logarithms.
- β =transmission coefficient or the strength of the field after absorption is accounted for.

Hence:

$$\delta + \beta = 1 \text{ or } 100 \text{ per cent,}$$

$$\delta = 1 - \beta.$$

To aid in the visualization of what is actually happening to the electrical and magnetic vectors, the following discussion is offered.

When a plane wave passes through an absorbing medium there occurs in its path of progress a change in amplitude and phase due to this absorption. These two processes are clearly differentiated in Figure 1. *A* illustrates a decrease in amplitude but no change in phase while *B* shows a change in phase with no change in amplitude. *C* combines *A* and *B* to give the resultant change in both phase and amplitude caused by absorption. Up to the present little attention has been given to the phase change, but it is beginning to be recognized as important in certain prospecting methods.

The graphic representations shown in Figure 1 are conveniently expressed by the equation:

$$H = H_0 e^{-\alpha l} = H_0 e^{-\alpha_1 l - j\alpha_2 l}.$$

This applies to either electrical or magnetic field intensity. H_0 is the initial amplitude of the vector at the surface, while H is the amplitude at any depth l , if the effect of distance is neglected. Thus, Figure 1, *A* shows the effect of making $\alpha_2 = 0$, while in Figure 1, *B*, $\alpha_1 = 0$. In Figure 1, *C*, both α_1 and α_2 have positive real values. The resulting locus is a spiral on a logarithmic cone.

Actually, the solution must be modified by a factor which accounts for the decrease in field intensity with increase in the distance from

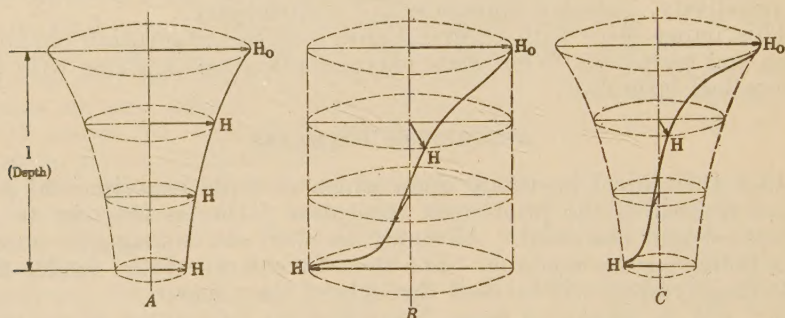


FIGURE 1.—Development of space and time diagrams of electromagnetic waves in absorptive medium

the exciting loop. Thus, for distances large compared to the energizing loop dimensions, the inverse cube of the distance enters.

$$H = H_0 \frac{r^3}{l^3} e^{-\alpha l}.$$

For smaller distances the field intensity along the axis of the exciting loop perpendicular to its plane is found to be:

$$H = H_0 \frac{r^3}{(l^2 + r^2)^{3/2}} e^{-\alpha l},$$

where r = radius of exciting loop.

The above equations are represented graphically in Figure 2. The effect of distance is seen to be the controlling factor, although absorption will in general reduce the amplitude of the vector still more.

It was the aim of this investigation to examine the importance of this absorption factor from an experimental viewpoint.

MAXWELL'S ABSORPTION FORMULA

The first, and probably still the foremost formula, was derived directly from Maxwell's fundamental equations of an electrodynamic field.

Neglecting for the time the change of phase caused by absorption, the transmission constant, β , is found to be:

$$\beta = e^{-\alpha l} = e^{-\frac{2\pi f k l}{c}},$$

from which the absorption factor, δ , becomes

$$\delta = 1 - \beta = 1 - e^{-\frac{2\pi f k l}{c}}. \quad (1)$$

Here,

$$k = \left[\frac{\mu}{2} \left(\sqrt{\epsilon^2 + \frac{4}{f^2 \rho^2}} - \epsilon \right) \right]^{\frac{1}{2}}.$$

All quantities are in electrostatic units (e. s. u.).

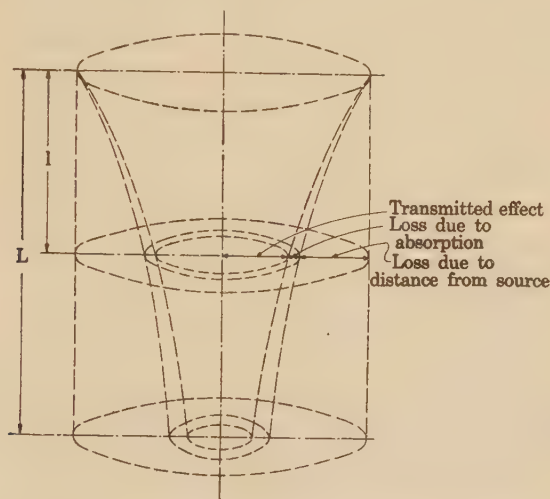


FIGURE 2.—Effect of distance and absorption on amplitude of waves

Because of its importance, the derivation of the above relation is now given, the conditions, as previously specified, being a plane wave in a homogeneous medium.

The fundamental equations of an electrodynamic field as given by Maxwell are:

$$-\frac{1}{c} \dot{B} = \text{curl } E, \quad (1-a)$$

$$\frac{1}{c} (\dot{D} + I) = \text{curl } H, \quad (1-b)$$

$$\text{div } B = 0, \quad (1-c)$$

$$\text{div } D = q. \quad (1-d)$$

For convenience the above equations are given in Heaviside-Lorentz units and the displacement, D , is defined as follows:

$$D = E + 4\pi P.$$

This is 4π times the usual value for displacement given in electrostatic units.

The following substitutions are now made:

$$D = \epsilon E \quad B = \mu H \quad I = \frac{E}{\rho}$$

Hence:

$$-\frac{\mu}{c}H = \text{curl } E, \quad (1-a')$$

$$\frac{1}{c}\left(\epsilon\dot{E} + \frac{E}{\rho}\right) = \text{curl } H, \quad (1-b')$$

$$\text{div } E = \frac{q}{\epsilon}, \text{ and} \quad (1-c')$$

$$\text{div } H = 0. \quad (1-d')$$

Multiplying (1-b') by μ and taking curl of both sides:

$$\frac{\mu\epsilon}{c} \text{curl } \dot{E} + \frac{\mu}{c\rho} \text{curl } E = \text{curl curl } B. \quad (1-e)$$

Now $\text{curl curl } B = \text{grad div } B - \Delta_2 B$,

$$= -\Delta_2 B,$$

and

$$\text{curl } \dot{E} = -\frac{1}{c} \ddot{B}.$$

Substituting in (1-e):

$$\frac{\mu\epsilon}{c^2} \ddot{B} + \frac{\mu}{c^2\rho} \dot{B} = \Delta_2 B. \quad (1-f)$$

Now consider the solution for the case of a plane wave in a homogeneous medium moving in a direction such that the two components of magnetic intensity, B_y and B_z , become zero. Then since B_x is a function of t and z alone, equation (1-f) reduces to:

$$\frac{\mu\epsilon}{c^2} \frac{d^2 B_x}{dt^2} + \frac{\mu}{c^2\rho} \frac{dB_x}{dt} = \frac{d^2 B_x}{dz^2}. \quad (1-g)$$

Assume a solution for this equation to be of the form:

$$B_x = B_0 e^{-AZ} e^{i(2\pi f t - DZ)}.$$

Substituting this in the differential equation and solving for the constant A :

$$A^2 = \frac{4\pi^2 f^2}{c^2} \left[\frac{\mu}{2} \left(\sqrt{\epsilon^2 + \frac{1}{4\pi^2 f^2 \rho^2}} - \epsilon \right) \right],$$

Changing to electrostatic units:

$$A^2 = \frac{4\pi^2 f^2}{c^2} \left[\frac{\mu}{2} \left(\sqrt{\epsilon^2 + \frac{4}{f^2 \rho^2}} - \epsilon \right) \right],$$

or

$$A = \frac{2\pi f}{c} K = \frac{2\pi f}{c} \left[\frac{\mu}{2} \left(\sqrt{\epsilon^2 + \frac{4}{f^2 \rho^2}} - \epsilon \right) \right]^{\frac{1}{2}}.$$

The absorption factor, δ , is plotted in Figure 3 for various values of resistivity and dielectric constant against the logarithm of the frequency. The depth l is taken as 124 feet. The resulting curves are extremely interesting for they give a clear picture of the various factors which govern the absorption.

In general, the curve consists of three distinct regions. In the first, the absorption is the same for both values of the dielectric

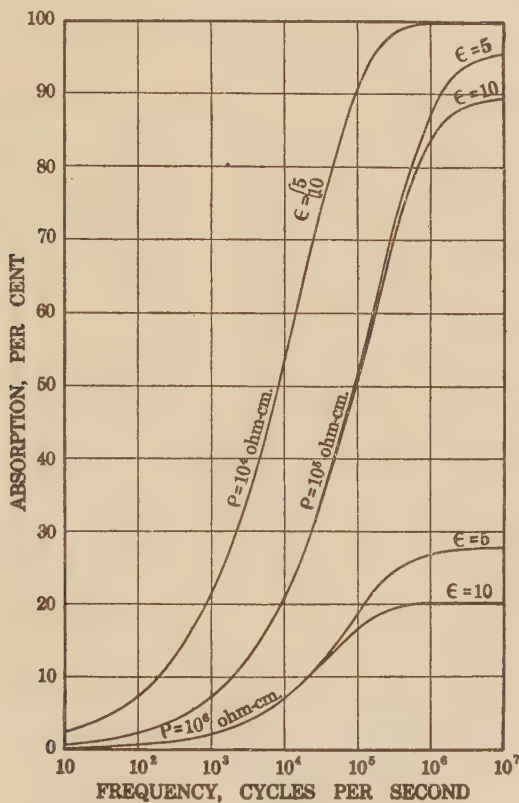


FIGURE 3.—Absorption based on Maxwell's relations

constant. Examining equation (1) it is seen that at low frequencies, for the values of ρ and ϵ considered, the dielectric constant may be neglected. This reduces the absorption factor to the relatively simple form:

$$\delta = 1 - e^{-\frac{2\pi}{c} \sqrt{\frac{l}{\rho}} \cdot Z}. \quad (2)$$

If ρ and z are constant this may be written:

$$\delta = 1 - e^{-K\sqrt{f}}, \text{ where } K = \frac{2\pi}{c\sqrt{\rho}} Z. \quad (2)$$

In the second region the effect of the dielectric constant begins to enter, and the two curves, corresponding to values of ϵ of 5 and 10,

separate. This portion of the curve corresponds to the exact relation, equation (1), where no simplifying approximations can be made.

Finally, the third section of the curve reveals that as the frequency is further increased, the absorption increases less and less rapidly, until a stage is reached where for all practical purposes the absorption

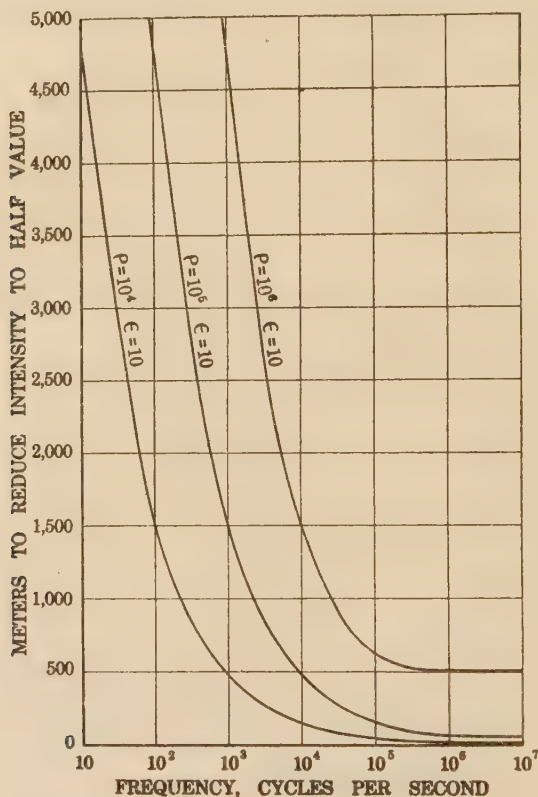


FIGURE 4.—Thickness reducing wave to one-half value by absorption

may be considered to be independent of the frequency. This may be shown by a study of equation (1). Expanding $\sqrt{\epsilon^2 + \frac{4}{f^2\rho^2}}$ by the binomial theorem:

$$\sqrt{\epsilon^2 + \frac{4}{f^2\rho^2}} = \epsilon + \frac{1}{2\epsilon} \frac{4}{f^2\rho^2} - \frac{1}{8\epsilon^3} \left(\frac{4}{f^2\rho^2} \right)^2 + \dots$$

For values of frequency sufficiently high, all terms of higher order than the second may be neglected. Then, considering ρ and z as constants, the absorption becomes:

$$\delta = 1 - e^{-K}, \text{ where } K = \frac{2\pi}{c\rho\sqrt{\epsilon}} Z. \quad (3)$$

Although equation (3) implies a value of absorption independent of frequency, it must be borne in mind that as the wave lengths approach

those of heat and light waves other factors enter which cause increased absorption. The values of ρ and ϵ become functions of the frequency, and resonance phenomena occur.

A discussion of this region will not be given, as it is beyond the scope of this paper.

As a matter of interest, Figure 4 shows the thickness of rock necessary to reduce the amplitude of the electromagnetic wave to one-half of its value, considering only absorption and neglecting distance effects.

ZENNECK'S ABSORPTION FORMULA

A second relation for absorption was originated by Zenneck and is given by Fleming. The case of plane waves traveling over a plane surface is considered, and the fact that changes of phase occur due to the conductivity and dielectric constant of the material over which the wave passes is demonstrated. This in turn gives rise to an elliptical polarization phenomena.

As in the previous case, the transmission factor is given as:

$$\beta = e^{-KZ}.$$

The absorption then becomes

$$\delta = 1 - \beta = 1 - e^{-KZ}, \quad (4)$$

where

$$K = \frac{\sqrt{2\pi}f}{c} \frac{\sqrt{\frac{1}{\rho^2} + \frac{f^2 \epsilon^2}{4}}}{\sqrt{\frac{1}{4\pi} \sqrt{\frac{1}{\rho^2} + \frac{f^2(\epsilon+1)^2}{4}}}} \sin\left(\phi_1 - \frac{\phi_2}{2} - \frac{\phi}{2}\right) \quad (5)$$

$$\phi = \frac{\pi}{2},$$

$$\phi_1 = \tan^{-1} \frac{f\epsilon\rho}{2}$$

$$\phi_2 = \tan^{-1} \frac{f(\epsilon+1)\rho}{2}.$$

The absorption is plotted against the logarithm of the frequency in Figure 5. As in the previous case, the depth z is taken as 124 feet and the same values of ρ and ϵ are used.

Here again the curve consists of three regions, the first corresponding to values of frequency where the dielectric constant may be neglected. This simplifies equation (4) to the form:

$$\delta = 1 - e^{-\frac{2\pi}{c} \sqrt{\frac{f}{\rho}} Z}. \quad (6)$$

This is seen to be the same as equation (2) of the Maxwell formula. Hence, in this region the two curves coincide.

In the second section of the curve the exact relation for absorption must be used.

When the frequency becomes sufficiently high, simplifications may be made, the trigonometric function may be expanded, and the resulting expression for the absorption becomes:

$$\delta = 1 - e^{-\frac{2\pi}{c\rho\sqrt{\epsilon+1}} Z}. \quad (7)$$

This expresses the fact that as in the Maxwell relation, δ becomes independent of the frequency. However, the same restrictions also apply, namely, that as optical wave lengths are approached, other factors occur which increase δ .

KING'S ABSORPTION FORMULA

Dr. L. V. King of McGill University at Montreal has investigated the problem of absorption from the standpoint of the mutual induc-

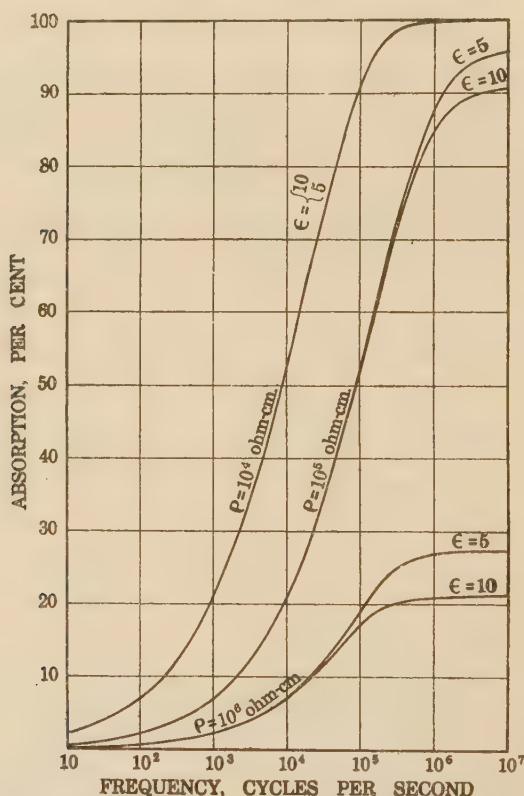


FIGURE 5.—Absorption based on Zenneck's relations

tance between two coils, separated by a medium of fairly low conductivity, such as rock.

When two horizontal coils are placed one above the other a distance z apart with their centers on the same axis perpendicular to their planes, if the medium between the coils is air, the mutual inductance is given as:

$$M_0 = \frac{2n n_1 A A^1}{Z^3},$$

where A and A^1 are areas of coils,

n and n_1 are turns on coils.

This relation only applies to cases where the distance of separation is great compared to the coil dimensions.

If a medium of low conductivity, such as rock, is introduced between the two coils, the value of the mutual inductance is modified by an attenuation factor, λ .

$$M = \lambda M_0. \quad (8)$$

Doctor King has found the factor λ to be a function of the resistivity of the material between the coils, its thickness, and the frequency of the electromagnetic field. The form of this function is given in a paper by Dr. A. S. Eve on the subject of absorption of electromagnetic waves by rocks as:

$$\lambda = \left(1 + \frac{2\pi}{c} \sqrt{\frac{f}{\rho}} Z\right) \left(1 + \frac{4\pi^2 f}{c^2 \rho} Z^2\right)^{1/2} e^{-\frac{2\pi}{c} \sqrt{\frac{f}{\rho}} Z}.$$

Here ρ is the resistivity in electrostatic units, as before. The effect of the dielectric constant has not been introduced. Based on the results obtained from Maxwell's equations and Zenneck's work it appears that in the cases studied this factor becomes an important one for frequencies over 10^5 cycles per second for the values of resistivity used.

King's formula is simply Maxwell's attenuation term where ϵ is neglected, with an additional multiplying factor which increases this term, and therefore reduces the value of the absorption.

At 10^5 cycles, this relation gives less than 1 per cent absorption, which is not in agreement with the previous formulas studied.

OLLENDORFF'S ABSORPTION FORMULA

Ollendorff gives for the attenuation term for plane electromagnetic waves:

$$\beta = e^{-\frac{2\pi}{c} \sqrt{\frac{f}{\rho}} Z},$$

and

$$\delta = 1 - e^{-\frac{2\pi}{c} \sqrt{\frac{f}{\rho}} Z}. \quad (9)$$

This is recognized at once to be Maxwell's relation with the dielectric constant neglected. In view of the previous discussion, this relation may be applied as long as ϵ is negligible. Beyond this region the exact form due to Maxwell applies.

SOMMERFELD'S ABSORPTION FORMULA

In his treatment of the transmission of wireless waves, Sommerfeld gives for the damping and transmission coefficient of the component of a plane wave normal to the plane separating two infinite media,

$$\alpha = \sqrt{K_1^2 - K_2^2} = \alpha_1 + j \alpha_2, \quad (10)$$

where

$$K^2 = \frac{4\pi^2 f^2 \epsilon \mu + j 2\pi f \mu \sigma}{c^2}.$$

In equation (10) the real part of α gives the damping while the imaginary part is the transmission coefficient.

If one medium is taken as air and the other rock the coefficient may be readily evaluated.

Thus:

$$\epsilon_1 = \mu_1 = \mu_2 = 1.$$

$$\sigma_2 = 0.$$

Dropping subscripts:

$$K_1^2 - K_2^2 = \frac{4\pi^2 f^2}{c^2} - \frac{4\pi^2 f^2 \epsilon}{c^2} - \frac{j}{c^2} \frac{2\pi f \sigma}{c^2}$$

$$\alpha = \sqrt{K_1^2 - K_2^2} = \sqrt{\left[\frac{4\pi^2 f^2}{c^2} (1 - \epsilon) \right]^2 + \left(\frac{2\pi f \sigma}{c^2} \right)^2} e^{+j \frac{1}{2} \tan^{-1} \frac{\sigma}{2\pi f (1 - \epsilon)}}.$$

Solving this relation for the real part which gives the damping term, α :

$$\alpha_1 = \frac{2\pi f}{c} \sqrt{(1 - \epsilon)^2 + \frac{\sigma^2}{4\pi^2 f^2}} \cos \left[\frac{1}{2} \tan^{-1} \frac{\sigma}{2\pi f (1 - \epsilon)} \right].$$

$$\alpha_1 = \frac{\sqrt{2}\pi f}{c} \sqrt{\sqrt{(1 - \epsilon)^2 + \frac{\sigma^2}{4\pi^2 f^2}} + (1 - \epsilon)}. \quad (11)$$

The above is expressed in Heaviside units. Changing to electrostatic units:

$$\alpha_1 = \frac{\sqrt{2}\pi f}{c} \sqrt{\sqrt{(1 - \epsilon)^2 + \frac{4\sigma^2}{f^2}} + (1 - \epsilon)}.$$

The transmission is then given by the term

$$\beta = e^{-\alpha_1 Z}.$$

For the absorption:

$$\delta = 1 - e^{-\alpha_1 Z}.$$

In certain regions the value of α_1 may be materially simplified without introducing an appreciable error by neglecting certain factors. For some values of frequency and conductivity $(1 - \epsilon)^2$ is negligible compared to $\frac{4\sigma^2}{f^2}$ and the exponent becomes simply:

$$\alpha_1 = \frac{2\pi}{c} \sqrt{f\sigma} = \frac{2\pi}{c} \sqrt{\frac{f}{\rho}}.$$

Again, for very high values of frequency $\frac{4\sigma^2}{f^2}$ becomes negligible compared to $(1 - \epsilon)^2$ and the exponent takes the form:

$$\alpha_1 = \frac{2\pi\epsilon}{c\sqrt{\epsilon - 1}} = \frac{2\pi}{\rho c \sqrt{\epsilon - 1}}.$$

From the above relations, it is apparent that Sommerfeld's formula gives values of absorption which agree with the Maxwell and Zenneck results up to the high frequency region and even there the differences are not of any appreciable magnitude.

COMPARISON OF ABSORPTION FORMULAS

A survey of the various relations for the absorption of plane electromagnetic waves in media having conductivity and a dielectric constant reveals several important facts.

Of the several relations discussed, three give practically the same results over the frequency range considered. These three, Maxwell's,

Zenneck's, and Sommerfeld's relations, give identical results in the region where the dielectric constant may be neglected. The greatest differences occur at the highest frequencies for high values of resistivity and dielectric constant, but even then there is only a 5 per cent discrepancy between them at 10^7 cycles per second. It has been pointed out before that Zenneck treated the case from the standpoint of the elliptical polarization of the wave, due to a dragging or phase change brought about in the vector by the action of the ground. However, the fundamental equations are Maxwell's, so that although the two expressions for the absorption are quite different in appearance they give almost identical results. The same applies to Sommerfeld's work. Ollendorff's relation is merely a simplification of Maxwell's formulas and holds only for low values of frequency where dielectric constant may be neglected.

King's formula does not agree at all with Maxwell's, Zenneck's, or Sommerfeld's. All values obtained from this relation for the frequency range used were extremely low.

Thus, the various relations fall into two distinct groups, one giving rather large values of absorption and the other extremely low values.

It is hoped that the experimental evidence will enable one to determine which group gives results nearest to the actual values over a range of frequencies from 20 to 10^8 cycles per second. At present the only frequency worked out in this manner is 500 cycles, and the experimental results favor decidedly the Maxwell, Zenneck, or Sommerfeld relations.

THEORY OF METHOD OF EXPERIMENTAL MEASUREMENT OF ABSORPTION

In computing the experimental values of absorption based upon observations taken in the field, the following procedure was used:

Consider two circular coils placed one above the other in parallel planes, their centers lying on the same straight line perpendicular to the coil faces. Also, let a medium having a dielectric constant and conductivity be introduced between the coils. This medium will cause absorption of an electromagnetic wave, in accordance with the previously discussed theories, and the result will be to give an apparent decrease in the coefficient of mutual inductance between the two coils over what it would be if only air intervened.

Now let one of the above coils correspond to the horizontal circular exciting loop placed on the surface of the ground and the other coil, a pick-up loop placed underground, with rock as the intervening medium. Then let:

M_0 = the apparent coefficient of mutual inductance between surface loop and pick-up coil.

M'_0 = coefficient of mutual inductance between surface loop and pick-up coil calculated from physical dimensions of the system assuming air to be the intervening medium.

I_1 = effective amperes in surface loop.

E_2 = effective volts induced in pick-up coil for rock as the intervening material with I_1 amperes in surface loop.

E_2' = effective volts induced in pick-up coil for air as the intervening material and I_1 amperes in surface loop.

$\omega = 2\pi f$, where f = frequency in cycles per second.

By definition, it is seen that M_0 corresponds to the value of the coefficient of mutual inductance which would be necessary to give the

field intensity actually observed underground if air were the only medium between the two coils.

Consider next a small circular loop of known dimensions and turns. Let this loop replace the surface loop, and let it have excitation ampere turns of magnitude to give the same induced electromotive force in the pick-up coil that was induced when the surface loop was used.

Call:

M_o = coefficient of mutual inductance between small primary coil and pick-up coil. Let this be called the calibration set-up.

i_1 = effective amperes in primary.

e_2 = effective volts induced in pick-up coil for i_1 amperes in primary coil.

Then by the laws of induction:

$$E_2 = I_1 \omega M_o \quad (1)$$

$$e_2 = i_1 \omega M_e \quad (2)$$

But by definition i_1 and M_e are chosen such that:

$$e_2 = E_2$$

therefore

$$I_1 \omega M_o = i_1 \omega M_e,$$

and

$$M_o = \frac{i_1}{I_1} M_e. \quad (3)$$

Now, if air were the only medium between the surface and pick-up coils the induced voltage in the latter would be:

$$E'_2 = I_1 \omega M_o^1. \quad (4)$$

At low frequencies induced voltage is directly proportional to the magnetic vector provided no magnetic saturation occurs and the frequency remains constant. Since air-cored wooden-coil frames were used, saturation could not have been reached, especially at the very low field intensities used, and the frequency was carefully kept constant. Therefore the absorption is by definition:

$$\delta = \frac{E_2^1 - E_2}{E_2^1}. \quad (5)$$

Substituting the relations expressed in the previous equations:

$$\delta = \frac{I_1 \omega M_o^1 - I_1 \omega M_o}{I_1 \omega M_o},$$

or

$$\delta = 1 - \frac{M_o}{M_o^1} = 1 - \frac{i_1}{I_1} \frac{M_e}{M_o^1}. \quad (6)$$

Thus, in making an experimental determination M_e and M_o^1 are calculated from the physical dimensions of the systems and the currents in the surface loop and the primary calibration loop are measured. From these four quantities δ is readily determined from equation (6).

This is in reality a substitution method, and its merit lies in the fact that an absolute measurement in the sense that actual field intensities are determined is not necessary.

It is now in order to devote a few lines to the derivations and discussion of relations for the coefficient of mutual inductance between two circular coils with planes parallel and not in the same plane.

FORMULAS FOR CALCULATION OF COEFFICIENT OF MUTUAL INDUCTANCE BETWEEN TWO CIRCULAR LOOPS WITH PLANES PARALLEL AND NOT IN THE SAME PLANE

Exact formula.—The fundamental relation for the coefficient of mutual inductance between two circuits is given as:

$$L_{12} = L_{21} = \iint \frac{\cos \psi}{r} ds_1 ds_2.$$

(For derivation see J. H. Jeans, *Electricity and Magnetism*, 5th ed., Cambridge, pp. 395–396.)

ds_1 and ds_2 are elements of the two circuits respectively.

ψ = angle between elements.

r = distance between elements.

Applied to the case of two horizontal turns of radii r_1 and r_2 in parallel planes whose centers lie along the same axis a distance L apart (fig. 6), this becomes:

$$L_{12} = \int_0^{2\pi} \int_0^{2\pi} \frac{\cos (\Theta_2 - \Theta_1) r_1 r_2 d\Theta_1 d\Theta_2}{[L^2 + r_1^2 + r_2^2 - 2r_1 r_2 \cos (\Theta_2 - \Theta_1)]^{1/2}} - \text{Let } \Theta_2 - \Theta_1 = \phi.$$

$$L_{12} = 2\pi \int_0^{2\pi} \frac{r_1 r_2 \cos \phi d\phi}{[L^2 + r_1^2 + r_2^2 - 2r_1 r_2 \cos \phi]^{1/2}}.$$

This equation is an elliptic integral, and its solution is made up of elliptic functions. A convenient form of solution is given in Bureau of Standards Circular 74 (*Radio Instruments and Measurements*, p. 274).

Approximate formula.—If the distance L is very large compared to r_1 and r_2 a somewhat simpler treatment of the problem exists.

The magnetic field at a distance L along the axis of one coil of a single turn due to unit current in that coil is given as:

$$H_o = \frac{2\pi r_1^2}{(L^2 + r_1^2)^{3/2}}.$$

For L large compared to r_1 this becomes:

$$H_o = \frac{2\pi r_1^2}{L^3} = \frac{2A_1}{L^3}.$$

$$A_1 = \pi r_1^2 = \text{area of coil 1.}$$

The total flux cut by the second coil will then be given as:

$$\Phi = H_o A_2 = \frac{2A_1 A_2}{L^3}.$$

And if n_1 and n_2 are the turns on two coils respectively, the total interlinkages of one for unit current the other will be:

$$M = \frac{2A_1A_2n_1n_2}{L^3}.$$

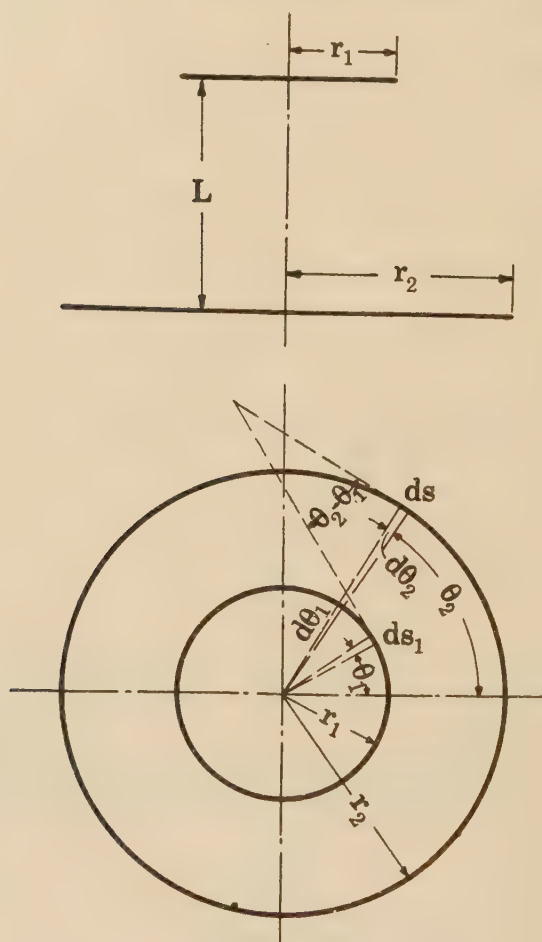


FIGURE 6.—Basis for computation of mutual induction

This is by definition the coefficient of mutual inductance between the two coils.

EXPERIMENTAL WORK

EARLY WORK

One of the first attempts made to work out the problem of the absorption of electromagnetic waves by rock on an experimental basis was carried out by Dr. A. S. Eve and Dr. D. A. Keys, both of McGill University. In their paper a series of experiments carried out in the Mount Royal Railroad Tunnel at Montreal is described.

Briefly, the work was done as follows: The receiving apparatus was set up in a wooden car and at certain convenient hours in the early morning this car was run into the tunnel, the locomotive then being withdrawn and the entrance blocked by a steel freight car. The latter precaution was intended to reduce transmission along the air column. Arrangements were made to have various frequency broadcasts from several near-by radio stations, the upper limit being 30 meters or 10^7 cycles per second. Observations were taken at the various frequencies throughout the length of the tunnel. The entire work was open to the serious criticism that some transmission of the waves undoubtedly occurred along the rails and wires running through the tunnel, as well as along the air column. In view of these disturbing features, most of the results are inconclusive. The only definite fact established as a result of the work was that at high frequencies (30 meters) no signals could be detected in the tunnel at a distance of more than a few yards from the entrance. This meant that at these high frequencies the absorption was very high.

Following this early work, the experiments described in this paper were carried out by Dr. A. S. Eve and Dr. D. A. Keys, of the Canadian Geological Survey, and Dr. F. W. Lee and J. Wallace Joyce, of the United States Bureau of Mines.

SITE OF WORK

The Mammoth Cave of Kentucky was selected as the site for the work, and it proved to be an ideal location. The cave is free from any metallic conductors leading in from the surface or in the cave itself, and with the exception of the main entrance all other openings of any appreciable size are water sealed. From the entrance the main cave is reached by a small winding passage about 750 feet long. The experimental stations were chosen sufficiently far from this entrance to eliminate any possible transmission along the passages. Any fields which were present undoubtedly came through the rock itself.

In so far as it affected the phenomena of absorption, the geology of the region is very simple. The strata are practically horizontal, having a northwest dip of 30 to 50 feet per mile. The plateau, beneath which the cave is situated, is capped with Cypress sandstone, to whose erosion-resisting qualities it owes its existence. Beneath the Cypress sandstone are found successive thick series of Renault-Paint Creek and St. Genevieve limestone, the former occurring above the latter. The two are sometimes combined and referred to as Mammoth Cave limestone. The presence of this limestone is indicated by caves, sink holes, and the almost complete absence of surface streams. Limestone is practically insoluble in pure water but is easily dissolved when the water contains carbon dioxide, as all surface waters do. It is the action of this water, percolating along cracks in the limestone, that eventually forms the passageways, rooms, and domes of the various caves.

A plan and sections of the cave are shown in Figure 7. The section on A-A' shows a layer of the Cypress sandstone extending from the surface to a depth of 101 feet. Below this formation is the massive Mammoth Cave limestone in which the cave itself is found. The location of the contact was obtained by tracing a very well-defined

layer of limestone from the entrance of the cave to the observational station which was established at the top of a fault formation known as the Corkscrew. This contact was also verified by earth-resistivity measurements.

The parts of the cave used in the work were surveyed, and points were located on the surface perpendicularly above the stations under-

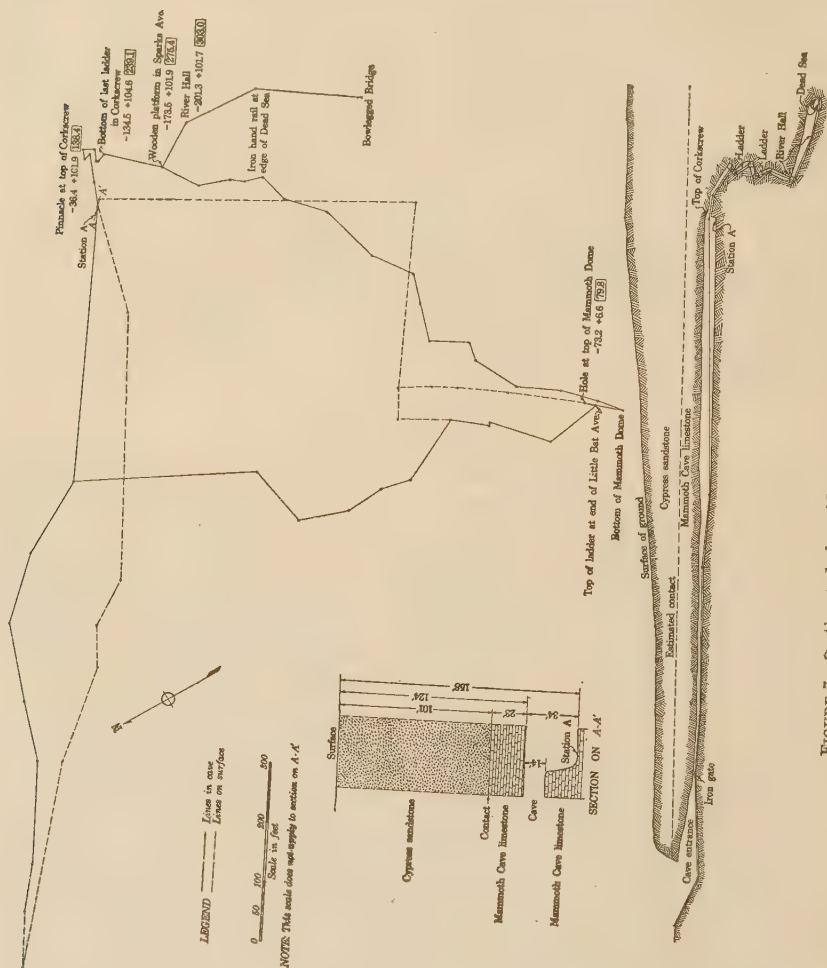


FIGURE 7.—Section and plan of Mammoth Cave of Kentucky

ground. The depth of the rock and the height of the cave ceiling from the floor were measured. At station A, which was at the top of the Corkscrew, the rock proved to be 124 feet thick, 101 feet being Cypress sandstone and 23 feet limestone. The cave ceiling at this point was 34 feet above the floor.

PRELIMINARY TESTS

The first step in the experimental work was to establish once and for all the agency of transmission of the electromagnetic waves. To

do this a station was set up in River Hall. A glance at Figure 7 will show that at this point the cave floor was 300 feet below the surface, and as the chamber was some 15 feet high there was about 285 feet of rock overhead at this point. The route in to River Hall was a twisting, turning path over 1,600 feet along, and in passing through the Corkscrew the way reversed itself several times, dropping over 100 feet at the same time. The extremely tortuous route taken removed entirely the possibility of any transmission along the air path. As has already been pointed out, no conductors of any description lead into the cave.

The result of all these precautions was to insure that any electromagnetic field which was present in the cave must of necessity have had to pass through the rock itself. Therefore, if such a field could be detected it would prove beyond all doubt that it had penetrated the rock. Once this point was established, a quantitative study could be made as to the relative magnitudes of the fields at various frequencies, with the idea of obtaining an experimental check of the absorption factor.

QUALITATIVE WORK

The month of June, 1929, was spent in carrying out qualitative tests in River Hall, and the ultimate result was to prove conclusively that waves of all frequencies up to 810 kilocycles, the highest frequency studied, penetrated the rock.

The lowest frequency investigated was 500 cycles. A small gasoline-driven alternator was used as a source of supply and three types of sending loops were tried. The dimensions and ampere turns of these various loops were as follows:

1. Vertical triangular loop; 120 square feet area, 58 ampere turns.
2. Vertical rectangular loop; 19.5 square feet area, 144 ampere turns.
3. Horizontal circular loop; 8,730 square feet area, 23 ampere turns.

With all of these loops the signals were detected in River Hall. The receiving apparatus consisted of a small coil of 400 turns with an area of about 2 square feet, connected to a 3-stage audio-frequency amplifier. The strength of the signals indicated a rather low value of absorption. With the horizontal sending loop the signals were audible without the amplifier, the head phones being connected directly across the receiving coil terminals.

To carry the work a step further the apparatus was moved to Mammoth Dome and the horizontal loop was energized. It was still possible to hear the signals without the amplifier. The distance through rock from sending loop to receiving coil was 790 feet at this point.

The next range of frequencies investigated was 20 to 110 kilocycles. A 50-watt oscillator supplied from the 500-cycle generator furnished the power to the loop, which consisted of a single horizontal turn of wire 100 feet in diameter. A Navy type R. E. low-frequency receiving set was used in the cave. It contained an antenna coupling unit, a 4-stage radio-frequency amplifier, a low-frequency receiver, and a tuned audio amplifier. Two types of antenna were tried. For the first, a single 300-foot length of wire was suspended in the cave. The second consisted of a horizontal loop of three turns 10 by 40 feet.

The signals were received using either of these for the aerial. In addition to these signals, ship-code signals were heard at various intervals throughout the test.

Finally, regular commercial broadcasting covering a frequency range of 650 to 810 kilocycles was investigated. A small portable 6-tube superheterodyne receiver with loop aerial was used for this work. Figures 8 and 9 show the results of the tests. The signals could only be heard in places where the rock covering was not too thick, although by using a 300-foot length of wire suspended in the cave, grounded at one end and coupled with the set by winding it three times around the small rectangular loop, stations were heard on the loud speaker in River Hall. In the two figures the short lines indicate the positions of the loop, and the long arrows drawn from the centers of the loop give the directions of the broadcasting stations. In this work no aerial other than the small loop supplied with the set was used.

The various positions of the loop in different parts of the cave for maximum signal strength from the same station seem to indicate that some refraction of the waves occurs due to transmission through the rock. These effects are quite pronounced along Little Bat Avenue (fig. 9).

It also appears from the results that the effect of the air path near the cave entrance is negligible, as far as transmission along that path

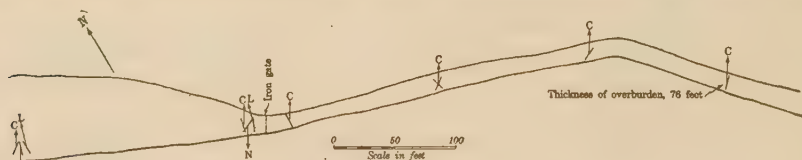


FIGURE 8.—Radio reception, starting from cave entrance

is concerned, doubtless because the broadcasting stations are practically at right angles to the direction of the cave.

In the ideal case of a spherical wave emitted from the broadcasting station the receiving loop should point directly to the station if no distortion of the wave occurred during transmission. However, intervening stretches of high ground and the effect of the electrical constants of the earth's surface, such as dielectric constant and resistivity, caused distortion, and in order to obtain a general idea of its magnitude the set was operated on the surface of the ground. The results are given in Figure 10.

This work at regular commercial broadcasting frequencies further substantiated the fact that transmission occurred through the rock itself. This was demonstrated by the fact that as the receiver was moved into the cave from the entrance, the signals diminished in strength until they were no longer audible, due to the increasing thickness of overburden. However, when the set was moved into Little Bat Avenue, the thickness of overburden decreased, and the signals were again heard, despite the fact that the cave entrance was over 2,000 feet away along a tortuous path.

This completed the qualitative work carried out in 1929, and the remainder of the paper is concerned with a series of quantitative tests made at 500 cycles per second during the month of June, 1930.

QUANTITATIVE WORK

Apparatus.—In order to establish the alternating electromagnetic field to be studied, a horizontal circular loop of five turns, 100 feet in diameter, was used. This loop was energized by a 750-watt gasoline-engine-driven 500-cycle radio alternator. The alternator voltage was

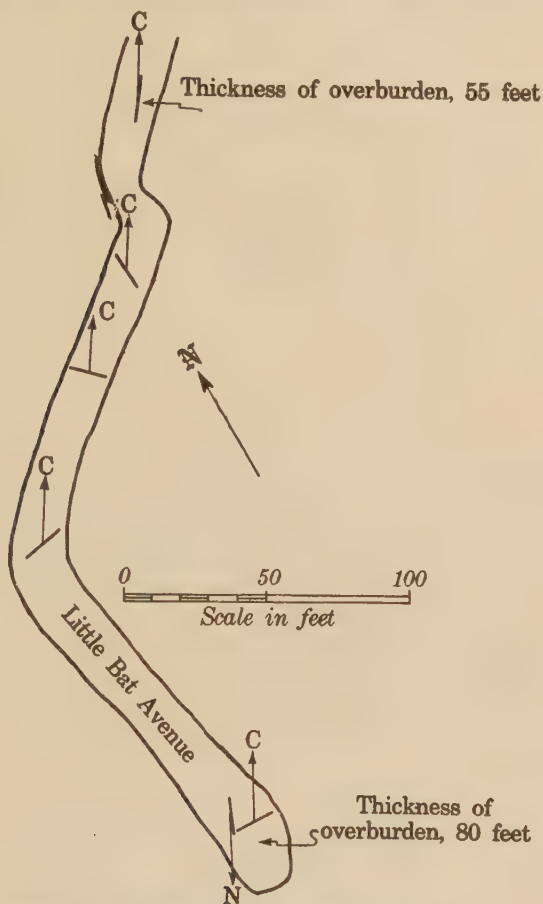


FIGURE 9.—Loop positions, radio reception starting from end of Little Bat Avenue over top of Mammoth Dome

too high, so a small railroad switch and signal transformer were used to reduce the potential to the loop.

In constructing the horizontal loop, upright stakes were set in the ground and the five turns were suspended from nails driven into the stakes. The turns were spaced 6 inches apart to form a vertical helix, avoiding any possibility of capacity-leakage currents either to ground or between turns. A Weston iron-vane ammeter was used to indicate the current in the loop.

The receiving circuit is shown in Figure 11. It consisted of a pick-up coil of 841 turns, 20 inches in diameter, tuned to resonance by means of a series capacity. The coil was mounted on a modified transit head

and could be rotated about both a horizontal and a vertical axis. The coil was wound on a wooden frame which had been boiled in paraffin, and the only metal parts used in its construction were two small brass trunnions for mounting purposes. A series resistance was placed in the circuit in case it became necessary to limit the current through the galvanometer. A 1-inch disk copper oxide rectifier was placed in series with the pick-up coil, and the direct current terminals of the

rectifier were connected to a Leeds and Northrop type P galvanometer with a sensitivity of about 10^{-8} amperes and a resistance of 120 ohms.

For calibration purposes a coil of 124 turns, 20 inches in diameter, was used as a primary. The arrangement is shown in Figure 12. This coil was connected to the alternator through the transformer and ammeter. In addition, a series capacity was provided for the purpose of tuning the coil to resonance.

In Figure 13 is shown the receiving apparatus as it was used. The pick-up coil is shown mounted on the transit base. The galvanometer is clamped to the plane table and the rectifier is placed in the white box on the table. The series capacity and resistance, both of the decade type built by General Radio Co., are seen on the case marked "Glass." The same set-up in the cave is shown in Figure 14. In this view the series resistance was omitted, and a switch was placed in the circuit so that the galvanometer zero could be checked from time to time.

The arrangement of the apparatus for calibration is shown in Figure 15. The primary coil with ammeter and transformer are shown in the foreground, while the receiving circuit is set up on the table in the background.

PROCEDURE

The first step in measuring the absorption experimentally was to set up the receiving apparatus in the cave and obtain a series of galvanometer readings corresponding to various values of current in the sending loop.

In making these observations the pick-up coil was set in such a position that the maximum induced voltage occurred. Since the field was not distorted by any conductors in the region this maximum was obtained when the plane of the coil was horizontal.

An operation schedule was always agreed upon between the party underground and the party operating the generator. Watches were synchronized, and in this manner a loop current reading was always taken on the surface to correspond to a galvanometer reading in the cave.

After a series of observations was taken underground the receiving apparatus was brought to the surface and calibrated. It was set up

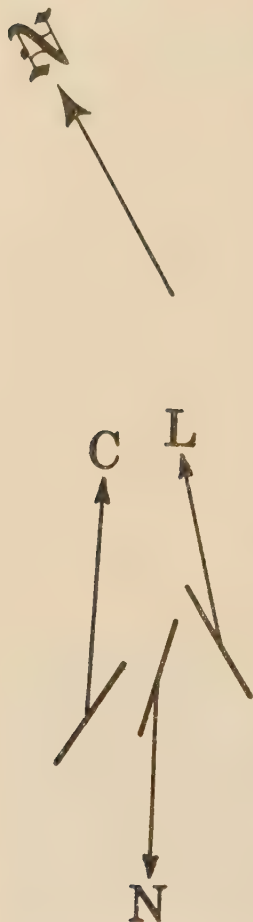


FIGURE 10.—Loop positions, radio reception on surface

exactly as it was used in the cave, and the small primary coil was connected to the supply transformer and placed so that its plane was parallel to the plane of the pick-up coil. The primary current was maintained at constant value throughout the run. The coils were placed sufficiently far apart to give a galvanometer reading slightly less than the smallest observed deflection in the cave. The primary was then moved closer to the pick-up coil by small steps, care being

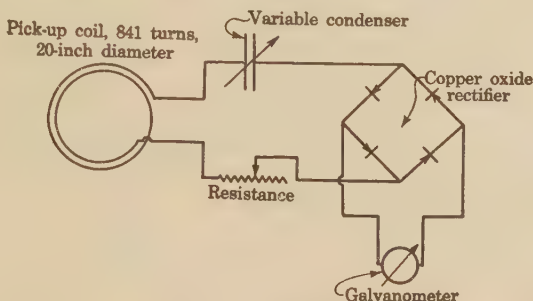


FIGURE 11.—Receiving circuit

taken to keep the planes of the two coils parallel, and a series of galvanometer readings was obtained which completely covered all readings taken underground. For each galvanometer reading there corresponded a definite value of mutual inductance (M_c) between the primary and pick-up coil.

In carrying on the experimental work a number of very important precautions had to be taken.

In the first place, it was found that the copper oxide rectifier possessed an appreciable temperature coefficient, and as a result of this it was necessary to keep it at constant temperature through all

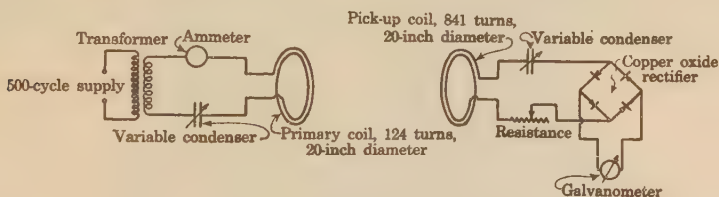


FIGURE 12.—Calibration arrangement

tests. An arbitrary temperature of 80° F. was chosen, and this was maintained in the cave by placing the rectifier in a heavy glass container placed in a bath of warm water. On the surface an ice bath replaced the warm one, and in this way the constant temperature was assured.

A second precaution was necessary when the receiving apparatus was used on the surface during calibration—to shade carefully all parts of the apparatus, especially the galvanometer, to avoid thermal electromotive forces. In addition, the rectifier had to be kept in a light-proof container, as the copper oxide was quite active photo-electrically.

All calculations and theoretical deductions are based upon the assumption of a pure sine wave. As a result the presence of any harmonic frequencies would introduce an error into the experiment. To be sure that such a discrepancy did not enter two calibration runs were made—one in which the primary was tuned, and a second in which it was untuned. The results of these runs indicated that any errors were within the range of experimental errors and could accordingly be neglected in the present work. No doubt the transformer acted as an effective choke to any frequencies higher than the fundamental, provided such frequencies were present at all.

EXPERIMENTAL RESULTS

A summary of the experimental data is given in Table 1. This shows the galvanometer deflections in the cave corresponding to definite currents in the horizontal loop.

TABLE 1.—*Experimental data*

Surface loop amperes	Net galvanometer deflections in cave, cm.
2.28	5.4
3.01	9.2
4.20	17.8

The calibration curves corresponding to the primary tuned and untuned are shown in Figure 16. The coils were never closer than 19 feet and accordingly the approximate formula for the mutual inductance was used.

From the data given in Table 1 and the calibration curves the following experimental values of absorption were obtained (Table 2), using the method of calculation previously outlined.

TABLE 2.—*Experimental values of absorption*

Surface loop amperes	Net galvanometer deflections in cave, cm.	Primary coil amperes for calibration	Mutual inductance in microhenrys for calibration (from fig. 13)	Absorption per cent (for 124 feet of rock)
2.28	5.4	1.20	2.13	3.9
3.01	9.2	1.20	2.77	5.4
4.20	17.8	1.20	3.85	5.7

In the above results the value of the coefficient of mutual inductance between surface loop and pick-up coil for air as the only medium (M_o^1) was computed by the exact formula to be 1.167 microhenrys.

The average value for the absorption, based on experimental results, is therefore 5.0 per cent for 124 feet of rock, 101 feet being sandstone and 23 feet limestone.

It is interesting to note that based on the experimental value of absorption, the field intensity in the cave for the lowest value of loop



FIGURE 13.—Receiving apparatus on the surface



FIGURE 14.—Receiving apparatus in the cave



FIGURE 15.—Calibration apparatus

current (2.28 amperes) was 1.43 times 10^{-4} gauss, while at the highest value (4.2 amperes) the intensity was 2.64 times 10^{-4} gauss. This meant that the voltages induced in the pick-up coil varied from 5.4 to 10.0 millivolts (effective values). These figures are based on a coil of 841 turns with a mean diameter of 20 inches.

DISCUSSION OF RESULTS

From the theoretical considerations given in the first part of this paper it is quite evident that at 500 cycles the dielectric constant plays a very small part in the absorption, and in all practical cases it may be neglected.

On the other hand the resistivity of the material determines what the absorption will be, the thickness of rock and frequency remaining

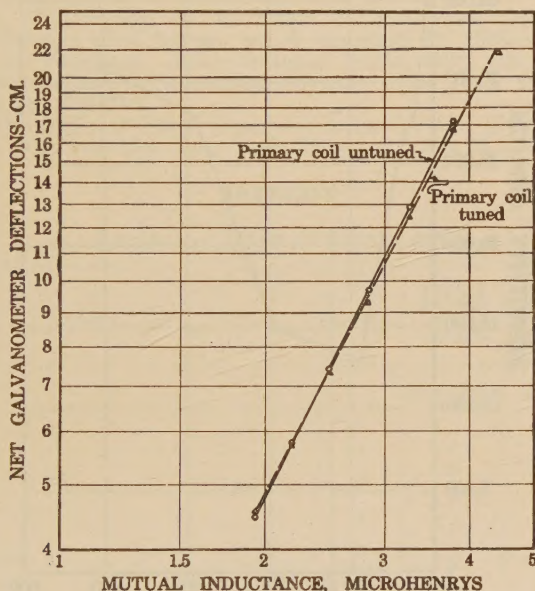


FIGURE 16.—Calibration curves

constant. Therefore, to calculate what the theoretical value of absorption should be, based on the previous formulas, an average value of resistivity must be known for the material overlying the cave.

These data were obtained by running a resistivity measurement directly over the section of the cave used. The results of this survey are given in the curve shown in Figure 17. From this curve the average value of resistivity is taken as 20,000 ohm centimeters and when substituted in either the Maxwell, Zenneck, or Sommerfeld formula, it gives for 124 feet of rock and 500 cycles an absorption of 11.2 per cent. If substituted in King's relation the absorption is found to be negligible.

From this it would appear that the experimental evidence favors Maxwell's, Zenneck's or Sommerfeld's relations, although the agreement is by no means close.

However, it must be borne in mind that all theoretical curves are based upon plane waves and homogeneous material, whereas in the experimental work the waves are not plane and the material is in the form of two layers. On the other hand, it seems natural to expect these factors to increase rather than to decrease the absorption, at least where the region investigated is so close to the source of the waves.

In conclusion, a great amount of progress has been made toward a successful solution to the problem of the absorption of electromagnetic waves by rock, and the fact that such waves actually penetrate

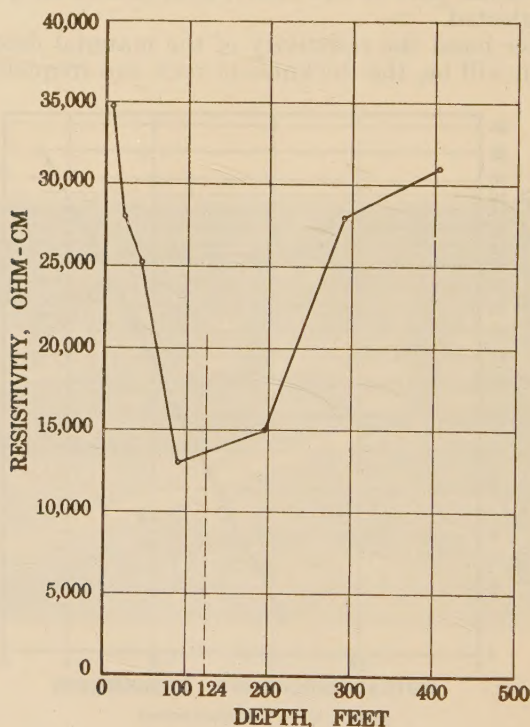


FIGURE 17.—Resistivity curve over "Corkscrew"

rock has been definitely established. A good start has been made toward the quantitative verification of absorption formula, and with the material contained in this paper as a foundation, additional experimental facts will be obtained in the near future which will no doubt provide a great additional amount of valuable information.

The results of this investigation show that a frequency of 500 cycles per second is well suited for electromagnetic prospecting. Absorption, although present, does not materially limit the applicability of this method.

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APPENDIX

TABLE 1.—Tables of ratios of units

Unit of—	Symbol	1 electro- magnetic unit equals—	1 electro- magnetic unit equals—	1 electro- static unit equals—
		<i>Electrostatic units</i>	<i>Heaviside units</i>	<i>Heaviside units</i>
Charge.....	q	c	$c\sqrt{4\pi}$	$\sqrt{4\pi}$
Current.....	i	c	$c\sqrt{4\pi}$	$\sqrt{4\pi}$
Electric intensity.....	E	$\frac{1}{c}$	$\frac{1}{c\sqrt{4\pi}}$	$\frac{1}{\sqrt{4\pi}}$
Potential.....	V	$\frac{1}{c}$	$\frac{1}{c\sqrt{4\pi}}$	$\frac{1}{\sqrt{4\pi}}$
Electric displacement.....	$D = \epsilon E$	c	$\frac{1}{\sqrt{4\pi}}$	$\frac{1}{\sqrt{4\pi}}$
Electric polarization.....	P	c	$c\sqrt{4\pi}$	$\sqrt{4\pi}$
Capacity.....	C	c^2	$4\pi c^2$	4π
Resistivity.....	ρ	$\frac{1}{c^2}$	$\frac{1}{4\pi c^2}$	$\frac{1}{4\pi}$
Conductivity.....	σ	c^2	$4\pi c^2$	4π
Self-inductance.....	L	$\frac{1}{c^2}$	$\frac{1}{4\pi c^2}$	$\frac{1}{4\pi}$
Magnetic intensity.....	H	c	$\frac{1}{\sqrt{4\pi}}$	$\frac{1}{c\sqrt{4\pi}}$
Intensity of magnetization.....	I	$\frac{1}{c}$	$\frac{1}{\sqrt{4\pi}}$	$\frac{1}{c\sqrt{4\pi}}$
Magnetic induction.....	B	$\frac{1}{c}$	$\frac{1}{\sqrt{4\pi}}$	$\frac{1}{c\sqrt{4\pi}}$
Dielectric constant.....	ϵ	1	1	1
Permeability.....	μ	1	1	1

TABLE 2.—*Ratios of practical units to electrostatic, electromagnetic, and Heaviside-Lorentz units*

Unit of—					
Charge.....	1 coulomb	$= 3 \times 10^9$	e. s. u. $= 10^{-1}$	e. m. u. $= 6\sqrt{\pi} \times 10^9$	H.-L. u.
Current.....	1 ampere	$= 3 \times 10^9$	e. s. u. $= 10^{-1}$	e. m. u. $= 6\sqrt{\pi} \times 10^9$	H.-L. u.
Potential.....	1 volt	$= \frac{1}{300}$	e. s. u. $= 10^8$	e. m. u. $= \frac{1}{600\sqrt{\pi}}$	H.-L. u.
Capacity.....	1 farad	$= 9 \times 10^{11}$	e. s. u. $= 10^{-9}$	e. m. u. $= 36\pi \times 10^{11}$	H.-L. u.
Resistivity.....	1 ohm-centimeter	$= \frac{1}{9 \times 10^{11}}$	e. s. u. $= 10^9$	e. m. u. $= \frac{1}{36\pi \times 10^{11}}$	H.-L. u.
Self-inductance.....	1 henry	$= \frac{1}{9 \times 10^{11}}$	e. s. u. $= 10^9$	e. m. u. $= \frac{1}{36\pi \times 10^{11}}$	H.-L. u.
Magnetic intensity.....	1 gauss	$= 3 \times 10^{10}$	e. s. u. $= 1$	e. m. u. $= \frac{1}{\sqrt{4\pi}}$	H.-L. u.
Dielectric constant.....	1 practical unit	$= 1$	e. s. u. $= 1$	e. m. u. $= 1$	H.-L. u.
Permeability.....	1 practical unit	$= 1$	e. s. u. $= 1$	e. m. u. $= 1$	H.-L. u.

